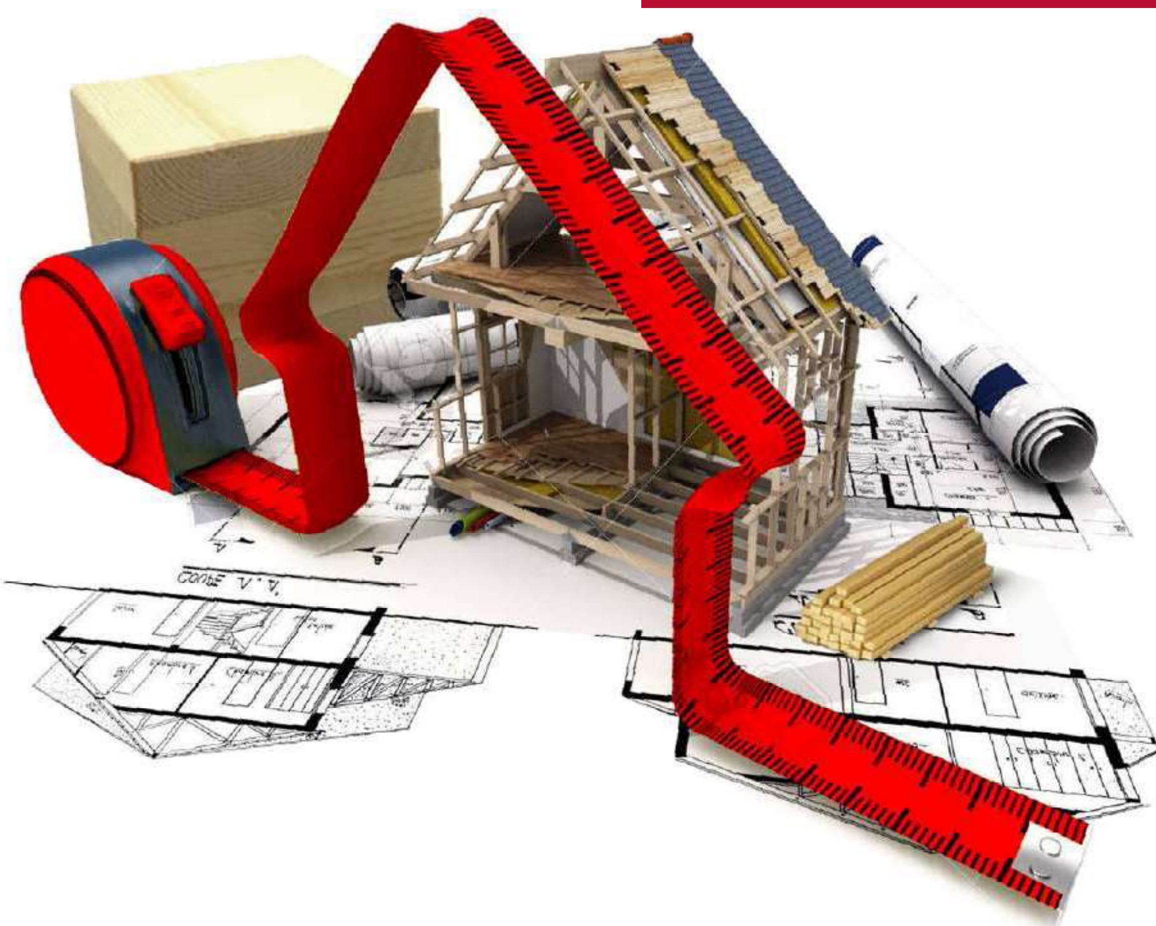
Section 10
Cross Laminated Timber Design Calculation Examples



Red Stag CLT Design Guide V1.4
July 2024

34. Overview

The Cross Laminated Timber (CLT) design examples in this section are provided to assist the market with the design and specification of Red Stag CLT. The technical examples provided have been developed based on the Canadian FPInnovation CLT Handbook, NZS 3603 Timber Structures Standard, NZS 1170 Structural Design Actions and the EN 1995-1-1 Eurocode 5 Design of Timber Structures (Refer to the *Table 36* below). This document is intended as a guide only (not a specification basis) to support in calculating and designing CLT members. Please refer to the relevant standards for further information to ensure that the project engineer, designer or specifier confirm the basis for each design to ensure it is fit for purpose and does not simply rely on the examples in this section.

Table 36: Referenced standards and documents utilised in the CLT floor design example.
The Red Stag CLT Floor Design Calculation Example has been developed in Conjunction with the Following Standards:
CLT Design Guide: FPInnovations CLT Handbook 2011, Chapter 3, Structural Design of CLT Elements. FPInnovations CLT Handbook 2011, Chapter 7, Vibration Performance of CLT Floors. Canadian CLT Handbook has been used as the primary design basis for Red Stag CLT to confirm the bending strength.
NZS 3603:1993: NZS 3603:1993 Timber Structures Standard is currently under review with an anticipated 2022 revision. Timber characteristics information from the New Zealand Timber Standard is used in Red Stag CLT floor design calculations.
AS/NZS 1170.1: AS/NZS 1170.1:2002 Structural design actions - Part 1: Permanent, imposed, and other actions. Permanent loads, imposed loads and load combinations from the New Zealand structural design action standard are used in Red Stag CLT design calculations.
EN 1995-1-1: EC 5: EN 1995-1-1:2004+A1:2008 - Eurocode 5: Design of timber structures. Vibration of the Red Stag CLT floor design example is calculated based on the recommended method in EN 1995-1-1:2004+A1:2008 - Eurocode 5, Section 7.5.

35.1 CLT Floor Panel Design – Longitudinal Direction

Calculation of the longitudinal members is based on the FPInnovation CLT design guide Mechanical jointed and simplified methods.

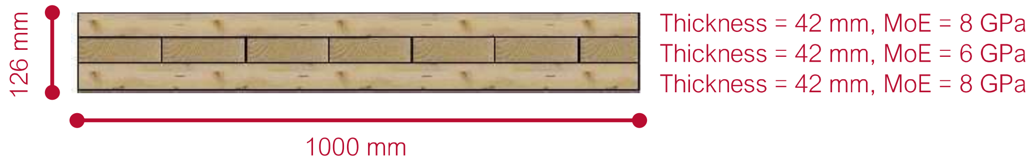


Figure 107: Red Stag CLT Panel Cross-Section

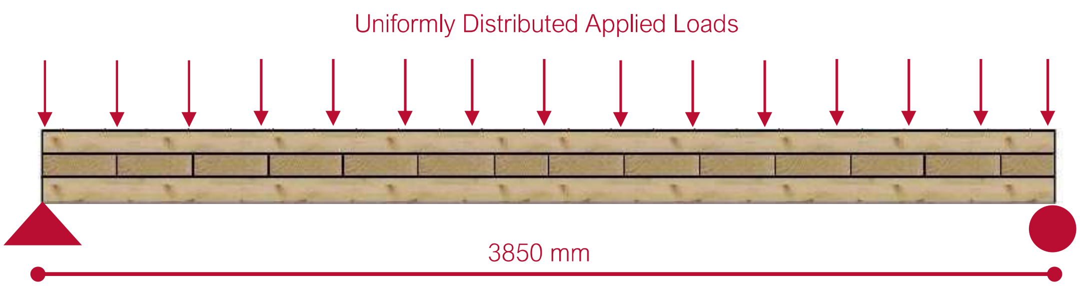


Figure 108: Red Stag CLT Panel Elevation

35.2 Assumption and Applied Loads:

Strength Reduction Factor (Ø) = 0.9 [36]

Bending Strength (F_b) = 14 MPa [36]

CLT Weight = 0.63 kPa - Calculated based on a Red Stag CLT density of 500 kg/m^3

Additional Dead Load = 0.5 kPa

Live Load = 2.0 kPa - Refer to AS/NZS 1170.1 [37]

35.3 Calculation of the Effective Bending Stiffness using the Mechanical Jointed Beam Theory (Gamma Method)

L = Span of panels = 3850 mm = 3.85 m

b = Width of the CLT panel = 1 m [37]

h_i = Thickness of board layers in direction of action [38]

h_1 = 42 mm

h_2 = 42 mm

$\overline{h_i}$ = Thickness of board layers in direction perpendicular to actions [38]

$\overline{h_1}$ = 42 mm

$A_i = b_i \times h_i$ [38]

A_1 = (42×1000) = 42000 mm^2

A_2 = (42×1000) = 42000 mm^2

$I_1 = \frac{b_i \times {h_i}^3}{12}$ [38]

$I_1 = \frac{b \times {h_1}^3}{12} = \frac{1000 \times 42^3}{12}$ = 6174000 mm^4

$I_2 = \frac{b \times {h_3}^3}{12} = \frac{1000 \times 42^3}{12}$ = 6174000 mm^4

E_1 = 8000 MPa [36]

E_2 = 8000 MPa [36]

$\gamma_1 = \frac{1}{1+\Pi^2 \times \frac{E_1 \times A_1}{L^2} \times \frac{h_1}{G_R \times b}}$ [38]

$\gamma_2 = \frac{1}{1+\Pi^2 \times \frac{E_3 \times A_3}{L^2} \times \frac{h_2}{G_R \times b}}$ [38]

where $\frac{h_i}{G_R \times b}$ = slip modulus due to shear deformation between layers and G_R = shear modulus perpendicular to the grain or rolling shear modulus [38].

E_0= MoE for longitudinal layers = 8000 MPa [36]	E_0= MoE for transvers layers = 6000 MPa [36]
E_{90} = 266.67 MPa	E_{90} = 200 MPa
G_0 = 500 MPa	G_0 = 375 MPa
G_R = 50 GPa [38]	G_R = 37.5 GPa [38]

L = span in mm (simple span; in direction of action //) [38]

$$\gamma_1 = \frac{1}{1+\frac{3.14^2\times 8000\times 42000}{3850^2}\times\frac{42}{37.5\times 1000}} = 0.89$$

$$\gamma_2 = \frac{1}{1+\frac{3.14^2\times 8000\times 42000}{3850^2}\times\frac{42}{37.5\times 1000}} = 0.89$$

$\overline{a}_1 = \frac{h_1}{2} + \frac{\overline{h_1}}{2}$ [38]

$\overline{a}_2 = \frac{h_2}{2} + \frac{\overline{h_1}}{2}$ [38]

$$\overline{a}_1 = \frac{42}{2} + \frac{42}{2} = 42 \text{ mm}$$

$$\overline{a}_2 = \frac{42}{2} + \frac{42}{2} = 42 \text{ mm}$$

$EI_{eff\,//} = \sum_{i=1}^{2}(E_i I_i + \gamma_i E_i A_i a_i^2)$ [38]

$EI_{eff\,//} = (E_1 I_1 + \gamma_1 E_1 A_1 a_1^2) + (E_2 I_2 + \gamma_2 E_2 A_2 a_2^2)$ [38]

$EI_{eff} = (8000\times 6174000+0.889\times 8000\times 42000\times 42^2)+$

$(8000\times 6174000+0.889\times 8000\times 42000\times 42^2) = 6\times 10^{11}+6\times 10^{11} = 1.152\times 10^{12}$ N.mm^2

$$I_{eff} = \frac{1.152\times 10^{12}}{8000} = 1.44\times 10^8 \text{ mm}^4$$

35.4 Calculation of Bending Strength using the Mechanically Jointed Beams Theory (Gamma Method)

$M_r = \emptyset \times F_b \times \frac{I_{eff}}{(\delta_1 a_1 + 0.5 h_1)}$ ($E_1 = E_2$) [38]

F_b = 14 MPa [36]

$\emptyset$ = 0.9 [36], [38]

$$M_r = 0.9\times 14\times\frac{1.44\times 10^8}{(0.89\times 42+0.5\times 42)}\times 10^{-6} = 31.12 \text{ kN.m}$$

35.5 Calculation of Bending Strength using the Simplified Method

$M_r = Ø \times F_b \times \frac{I_{eff}}{0.5h_1}$ [38]

$F_b = 14$ [36]

$Ø = 0.9$ [36], [38]

$M_r = 0.9 \times 14 \times \frac{1.44 \times 10^8}{0.5 \times 126} \times 10^{-6} = 28.81$ kN.m

35.6 Calculation of Applied Bending Moment

$M^* = \frac{(1.2 \times \text{Dead Load} + 1.5 \times \text{Live Load}) \times Span^2}{8}$ [38], [39]

$M^* = \frac{(1.35 \times \text{Dead Load}) \times Span^2}{8}$ [38], [39]

Dead Load = CLT Weight + Additional Dead Load

CLT Weight = CLT Thickness × Timber Weight = 126 × (5/1000) = 0.63 kPa

Additional Dead Load = 0.5 kPa.

Live Load = 2 kPa on Floor

$M^* = \frac{(1.2 \times (0.63 + 0.5) + 1.5 \times 2) \times 3.85^2}{8} = 8.07$ kN.m

$M^* = \frac{(1.35 \times (0.63 + 0.5)) \times 3.85^2}{8} = 3.14$ kN.m

35.7 Bending Capacity Check

$M_{r\ \text{Mechanical jointed method}}$ = 31.12 kN.m ≥ M^*= 8.07 kN.m √ ok

$M_{r\ \text{Mechanical jointed method}}$ = 31.12 kN.m ≥ M^*= 3.14 kN.m √ ok

$M_{r\ \text{Simplified method}}$ = 28.81 kN.m ≥ M^*= 8.07 kN.m √ ok

$M_{r\ \text{Simplified method}}$ = 28.81 kN.m ≥ M^*= 3.14 kN.m √ ok

35.8 Deflection Check

$$\Delta_{\text{CLT Deflection}} = \frac{5\times(\text{Uniformly Distributed Applied Load})\times Span^4}{(384\times(EI_{eff})}$$ [38], [39]

$$\Delta_{\text{CLT Deflection}} = \frac{5\times((\text{CLT Weight} + \text{Additional Dead Load})+0.4\times\text{Live Load})\times 3850^4}{(384\times(EI_{eff}))}$$

$$\Delta_{\text{CLT Deflection}} = \frac{5\times((0.63+0.5)+0.4\times 2)\times 3850^4}{(384\times(1.152\times 10^{12})} = 4.79 \text{ mm}$$

Creep Factor (K_2) = 2

Long term deflection = 4.79 × 2 = 9.58 mm → long term deflection

Long term deflection = 9.58 ≤ $\Delta^* = \frac{3850}{400}$ = 9.625 mm √ ok

35.9 Vibration Check

$$f = \frac{3.142}{2L^2}\sqrt{\frac{(EI)^{1m}_{eff}}{\rho A}}$$ [40]

ρ × A = m = is the mass per unit area in kg/m^2.

L = is the floor span in m.

m = linear mass of the CLT for a 1-m wide panel (kg/m).

EI_{eff} = effective bending stiffness.

$$f = \frac{3.142}{2\times 3.85^2}\sqrt{\frac{1.152\times 10^{12}}{500\times(1\times\frac{126}{1000})}} = 14.33 \geq 8\text{Hz}$$ √ ok

36.1 CLT Floor Panel Design – Longitudinal Direction

Calculation of the longitudinal members is based on the FPInnovation CLT design guide Mechanical jointed and simplified methods.

Figure 109: Red Stag CLT Panel Cross-Section

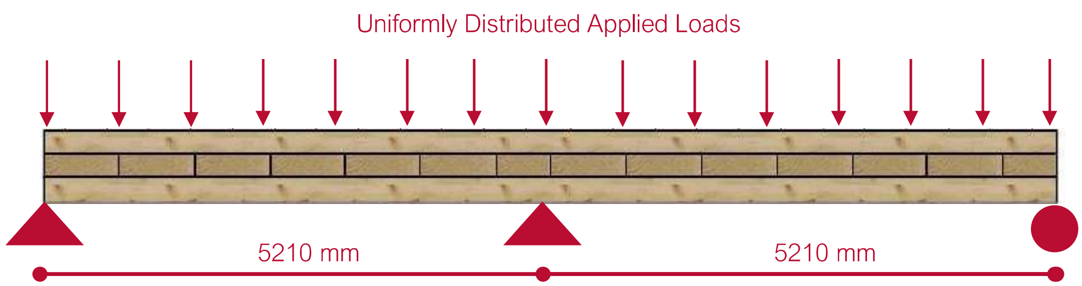


Figure 110: Red Stag CLT Panel Elevation

36.2 Assumption and Applied Loads:

Strength Reduction Factor (Ø) = 0.9 [36]

Bending Strength (F_b) = 14 MPa [36]

CLT Weight = 0.63 kPa - Calculated based on a Red Stag CLT density of 500 kg/m^3

Additional Dead Load = 0.5 kPa

Live Load = 2.0 kPa - Refer to AS/NZS 1170.1 [37]

36.3 Calculation of the Effective Bending Stiffness using the Mechanical Jointed Beam Theory (Gamma Method)

L = Span of panels = 5210 mm = 5.21 m

b = Width of the CLT panel = 1 m [38]

h_i = Thickness of board layers in direction of action [38]

h_1 = 42 mm

h_2 = 42 mm

$\overline{h}_i$ = Thickness of board layers in direction perpendicular to actions [38]

$\overline{h}_1$ = 42 mm

$A_i = b_i \times h_i$ [38]

$A_1 = (42\times1000) = 42000\ mm^2$

$A_2 = (42\times1000) = 42000\ mm^2$

$I_1 = \frac{b_i \times {h_i}^3}{12}$ [38]

$I_1 = \frac{b \times {h_1}^3}{12} = \frac{1000\times42^3}{12} = 6174000\ mm^2$

$I_2 = \frac{b \times {h_3}^3}{12} = \frac{1000\times42^3}{12} = 6174000\ mm^2$

E_1 = 8000 MPa [36]

E_2 = 8000 MPa [36]

$\gamma_1 = \frac{1}{1+\Pi^2\times\frac{E_1\times A_1}{L^2}\times\frac{\overline{h}_1}{G_R\times b}}$ [38]

$\gamma_2 = \frac{1}{1+\Pi^2\times\frac{E_3\times A_3}{L^2}\times\frac{\overline{h}_2}{G_R\times b}}$ [38]

where $\frac{\overline{h}_i}{G_R\times b}$ = slip modulus due to shear deformation between layers and G_R = shear modulus perpendicular to the grain or rolling shear modulus [38].

E_0= MoE for longitudinal layers = 8000 MPa [36]

E_{90} = 266.67 MPa

G_0 = 500 MPa

G_R = 50 GPa [38]

E_0= MoE for transvers layers = 6000 MPa [36]

E_{90} = 200 MPa

G_0 = 375 MPa

G_R = 37.5 GPa [38]

L = span in mm (simple span; in direction of action //) [38]

$$\gamma_1 = \frac{1}{1+\frac{3.14^2\times 8000\times 42000}{5210^2}\times\frac{42}{37.5\times 1000}} = 0.936$$

$$\gamma_2 = \frac{1}{1+\frac{3.14^2\times 8000\times 42000}{5210^2}\times\frac{42}{37.5\times 1000}} = 0.936$$

$a_1= \frac{h_1}{2} + \frac{\overline{h_1}}{2}$ [38]

$a_2= \frac{h_2}{2} + \frac{\overline{h_1}}{2}$ [38]

$a_1= \frac{42}{2} + \frac{42}{2}$ = 42 mm

$a_2= \frac{42}{2} + \frac{42}{2}$ = 42 mm

$EI_{eff\,//} = \sum_{i=1}^{2}(E_iI_i + \gamma_i E_i A_i {a_i}^2)$ [38]

$EI_{eff\,//} = (E_1I_1 + \gamma_1E_1A_1{a_1}^2) + (E_2I_2 + \gamma_2E_2A_2{a_2}^2)$ [38]

EI_{eff} $=(8000\times6174000+0.936\times8000\times42000\times42^2)+ (8000\times6174000+0.936\times8000\times42000\times42^2)$

$= 6\times10^{11}+6\times10^{11}$

$= 1.208\times10^{12}$ N.mm^2

$I_{eff} = \frac{1.208\times 10^{12}}{8000} = 1.51 \times 10^8$ mm^4

36.4 Calculation of Bending Strength using the Mechanically Jointed Beams Theory (Gamma Method)

$M_r = \varnothing\times F_b\times\frac{I_{eff}}{(\delta_1 a_1+0.5h_1)}$ $(E_1 = E_2)$ [38]

F_b = 14 MPa [36]

$\varnothing$ = 0.9 [36], [38]

$M_r = 0.9 \times 14 \times\frac{1.51\times 10^8}{(0.936\times 42+0.5\times 42)}\times 10^{-6}$ = 31.55 kN.m

36.5 Calculation of Bending Strength using the Simplified Method

$M_r = Ø \times F_b \times \frac{I_{eff}}{0.5h_1}$ [38]

$F_b = 14$ [36]

$Ø = 0.9$ [36], [38]

$M_r = 0.9 \times 14 \times \frac{1.51 \times 10^8}{0.5 \times 126} \times 10^{-6} = 30.21$ kN.m

36.6 Calculation of Applied Bending Moment

$M^* = \frac{(1.2 \times \text{Dead Load} + 1.5 \times \text{Live Load}) \times Span^2}{8}$ [38], [39]

$M^* = \frac{(1.35 \times \text{Dead Load}) \times Span^2}{8}$ [38], [39]

Dead Load = CLT Weight + Additional Dead Load

CLT Weight = CLT Thickness × Timber Weight = 126 × (5/1000) = 0.63 kPa.

Additional Dead Load = 0.5 kPa.

Live Load = 2 kPa on Floor

$M^* = \frac{(1.2 \times (0.63+0.5) + 1.5 \times 2) \times 5.21^2}{8} = 14.78$ kN.m

$M^* = \frac{(1.35 \times (0.63+0.5)) \times 5.21^2}{8} = 5.75$ kN.m

36.7 Bending Capacity Check

$M_{r\ \text{Mechanical jointed method}}$ = 31.55 kN.m ≥ M^*= 14.78 kN.m √ ok

$M_{r\ \text{Mechanical jointed method}}$ = 31.55 kN.m ≥ M^*= 5.75 kN.m √ ok

$M_{r\ \text{Simplified method}}$ = 30.21 kN.m ≥ M^*= 14.78 kN.m √ ok

$M_{r\ \text{Simplified method}}$ = 30.21 kN.m ≥ M^*= 5.75 kN.m √ ok

36.8 Deflection Check

$\Delta_{\text{CLT Deflection}} = \frac{5\times(\text{Uniformly Distributed Applied Load})\times Span^4}{(384\times(EI_{eff}\ )}$ [38], [39]

$\Delta_{\text{CLT Deflection}} = \frac{5\times((\text{CLT Weight + Additional Dead Load })+0.4\times\text{Live Load })\times 3850^4}{(384\times(EI_{eff}\ ))}$

$\Delta_{\text{CLT Deflection}} = \frac{5\times((0.63+0.5)+0.4\times 2)\times 5210^4}{(384\times(1.208\times 10^{12}\ )}$ = 15.32 mm

Creep Factor (K_2) = 2

Long term deflection = $\frac{15.32}{2.4}\times 2$ = 12.77 mm → Long term deflection

Long term deflection = 12.77 mm **≤** $\Delta^* = \frac{5210}{400}$ = 13.025 mm √ ok

36.9 Vibration Check

$f = \frac{3.142}{2L^2}\sqrt{\frac{(EI)_{eff}^{1m}}{\rho\ A}}$ [40]

ρ × A = m = is the mass per unit area in kg/m^2.

L = is the floor span in m.

m = linear mass of the CLT for a 1-m wide panel (kg/m).

EI_{eff} = effective bending stiffness.

$f = \frac{3.142}{2\times 5.21^2}\sqrt{\frac{1.208\times 10^{12}}{500\ \times(1\times\frac{126}{1000})}}$ = 8.02 **≥** 8Hz √ ok

37.1 CLT Floor Panel Design – Longitudinal Direction

Calculation of the longitudinal members is based on the FPInnovation CLT design guide Mechanical jointed and simplified methods.

Figure 111: Red Stag CLT Panel Cross-Section

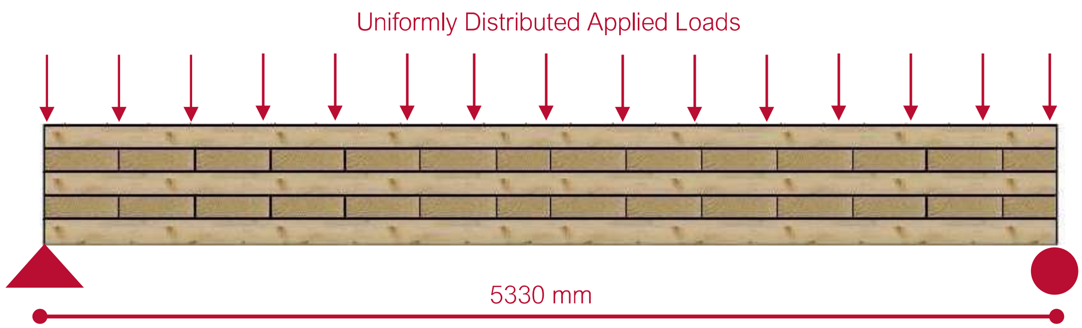


Figure 112: Red Stag CLT Panel Elevation

37.2 Assumption and Applied Loads:

Strength Reduction Factor (Ø) = 0.9 [36]

Bending Strength (F_b) = 14 MPa [36]

CLT Weight = 0.63 kPa - Calculated based on a Red Stag CLT density of 500 kg/m^3.

Additional Dead Load = 0.5 kPa

Live Load = 3.0 kPa - Refer to AS/NZS 1170.1 [37]

37.3 Calculation of the Effective Bending Stiffness using the Mechanical Jointed Beam Theory (Gamma Method)

L = Span of panels = 5330 mm = 5.33 m

b = Width of the CLT panel = 1 m [38]

h_i = Thickness of board layers in direction of action [38]

h_1 = 42 mm

h_2 = 42 mm

h_3 = 42 mm

$\overline{h_i}$ = Thickness of board layers in direction perpendicular to actions [38]

$\overline{h_1}$ = 42 mm

$\overline{h_2}$ = 42 mm

$A_i = b_i \times h_i$ [38]

$A_1 = (42\times1000) = 42000\ mm^2$

$A_2 = (42\times1000) = 42000\ mm^2$

$A_3 = (42\times1000) = 42000\ mm^2$

$I_i = \frac{b_i \times {h_i}^3}{12}$ [38]

$I_1 = \frac{b \times {h_1}^3}{12} = \frac{1000\times42^3}{12} = 6174000\ mm^4$

$I_2 = \frac{b \times {h_3}^3}{12} = \frac{1000\times42^3}{12} = 6174000\ mm^4$

$I_3 = \frac{b \times {h_3}^3}{12} = \frac{1000\times42^3}{12} = 6174000\ mm^4$

E_1 = 8000 MPa [36]

E_2 = 8000 MPa [36]

E_3 = 8000 MPa [36]

$\gamma_2 = 1$ [38]

$\gamma_1 = \frac{1}{1+\Pi^2\times\frac{E_1\times A_1}{L^2}\times\frac{\overline{h_1}}{G_R\times b}}$ [38]

$\gamma_3 = \frac{1}{1+\Pi^2 \times \frac{E_3 \times A_3}{L^2} \times \frac{\overline{h_2}}{G_R \times b}}$ [38]

where $\frac{h_i}{G_R \times b}$ = slip modulus due to shear deformation between layers and G_R = shear modulus perpendicular to the grain or rolling shear modulus [38].

E_0= MoE for longitudinal layers = 8000 MPa [36]	E_0= MoE for transvers layers = 6000 MPa [36]
E_{90} = 266.67 MPa	E_{90} = 200 MPa
G_0 = 500 MPa	G_0 = 375 MPa
G_R = 50 GPa [38]	G_R = 37.5 GPa [38]

L = span in mm (simple span; in direction of action //) [38]

$\gamma_2 = 1$ [38]

$\gamma_1 = \frac{1}{1+\frac{3.14^2 \times 8000 \times 42000}{5330^2} \times \frac{42}{37.5 \times 1000}} = 0.884$

$\gamma_3 = \frac{1}{1+\frac{3.14^2 \times 8000 \times 42000}{5330^2} \times \frac{42}{37.5 \times 1000}} = 0.884$

$\overline{a_1} = \frac{h_1}{2} + \overline{h_1} + \frac{h_2}{2}$ [38]

$\overline{a_2} = \frac{h_2}{2} + \overline{h_2} + \frac{h_3}{2}$ [38]

$\overline{a_1} = \frac{42}{2} + 42 + \frac{42}{2} = 82$ mm

$\overline{a_2} = \frac{42}{2} + 42 + \frac{42}{2} = 82$ mm

$EI_{eff //} = \sum_{i=1}^{2} (E_i I_i + \gamma_i E_i A_i {a_i}^2)$ [38]

$EI_{eff //} = (E_1 I_1 + \gamma_1 E_1 A_1 {a_1}^2) + I_2 + (E_3 I_3 + \gamma_3 E_3 A_3 {a_3}^2)$ [38]

$EI_{eff} = 4.34 \times 10^{12}$ N.mm^2

$I_{eff} = \frac{4.34 \times 10^{12}}{8000} = 5.43 \times 10^8$ mm^4

37.4 Calculation of Bending Strength using the Mechanically Jointed Beams Theory (Gamma Method)

$M_r = \varnothing \times F_b \times \frac{I_{eff}}{(\delta_1 a_1 + 0.5 h_1)}$ ($E_1 = E_2$) [38]

F_b = 14 MPa [36]

Ø = 0.9 [36], [38]

$M_r = 0.9 \times 14 \times \frac{5.43 \times 10^8}{(0.884 \times 84 + 0.5 \times 42)} \times 10^{-6} = 71.76$ kN.m

37.5 Calculation of Bending Strength using the Simplified Method

$M_r = \varnothing \times F_b \times \frac{I_{eff}}{0.5 h_1}$ [38]

F_b = 14 [36]

Ø = 0.9 [36], [38]

$M_r = 0.9 \times 14 \times \frac{5.43 \times 10^8}{0.5 \times 210} \times 10^{-6} = 28.81$ kN.m

37.6 Calculation of Applied Bending Moment

$M^* = \frac{(1.2 \times \text{Dead Load} + 1.5 \times \text{Live Load}) \times Span^2}{8}$ [38], [39]

$M^* = \frac{(1.35 \times \text{Dead Load}) \times Span^2}{8}$ [38], [39]

Dead Load = CLT Weight + Additional Dead Load

CLT Weight = CLT Thickness × Timber Weight = 210 × (5/1000) = 1.08 kPa.

Additional Dead Load = 0.5 kPa.

Live Load = 2 kPa on Floor

$M^* = \frac{(1.2 \times (1.08 + 0.5) + 1.5 \times 3) \times 5.33^2}{8} = 22.59$ kN.m

$M^* = \frac{(1.35 \times (1.08 + 0.5)) \times 5.33^2}{8} = 8.26$ kN.m

37.7 Bending Capacity Check

$M_{r\ \text{Mechanical jointed method}}$ = 71.76 kN.m ≥ M^*= 22.59 kN.m √ ok

$M_{r\ \text{Mechanical jointed method}}$ = 71.76 kN.m ≥ M^*= 8.26 kN.m √ ok

$M_{r\ \text{Simplified method}}$ = 65.13 kN.m ≥ M^*= 22.59 kN.m √ ok

$M_{r\ \text{Simplified method}}$ = 65.13 kN.m ≥ M^*= 8.26 kN.m √ ok

37.8 Deflection Check

$$\Delta_{\text{CLT Deflection}} = \frac{5\times(\text{Uniformly Distributed Applied Load})\times Span^4}{(384\times(EI_{eff}))}$$ [38], [39]

$$\Delta_{\text{CLT Deflection}} = \frac{5\times((\text{CLT Weight + Additional Dead Load})+0.4\times\text{Live Load})\times5330^4}{(384\times(EI_{eff}))}$$

$$\Delta_{\text{CLT Deflection}} = \frac{5\times((1.08+0.5)+0.4\times3)\times5330^4}{(384\times(4.34\times10^{12}))} = 6.66 \text{ mm}$$

Creep Factor (K_2) = 2

Long term deflection = 6.66 × 2 = 13.31 mm → long term deflection

Long term deflection = 13.31 ≤ $\Delta^* = \frac{5330}{400}$ = 13.325 mm √ ok

37.9 Vibration Check

$$L \leq 0.11\,\frac{\left(\frac{(EI)_{eff}}{10^6}\right)^{0.293}}{m^{0.123}}$$ [41]

L = vibration -controlled span limit in m.

L = is the floor span in m.

m = linear mass of the CLT for a 1-m wide panel (kg/m).

EI_{eff} = effective bending stiffness.

$$L \leq 0.11\,\frac{\left(\frac{4.34\times10^{12}}{10^6}\right)^{0.293}}{(1.0\times0.210\times500)^{0.123}} = 5.47 \text{ m} \geq 5.33 \text{ m}$$ √ ok

38.1 CLT Floor Panel Design – Longitudinal Direction

Calculation of the longitudinal members is based on the FPInnovation CLT design guide Mechanical jointed and simplified methods.

Figure 113: Red Stag CLT Panel Cross-Section

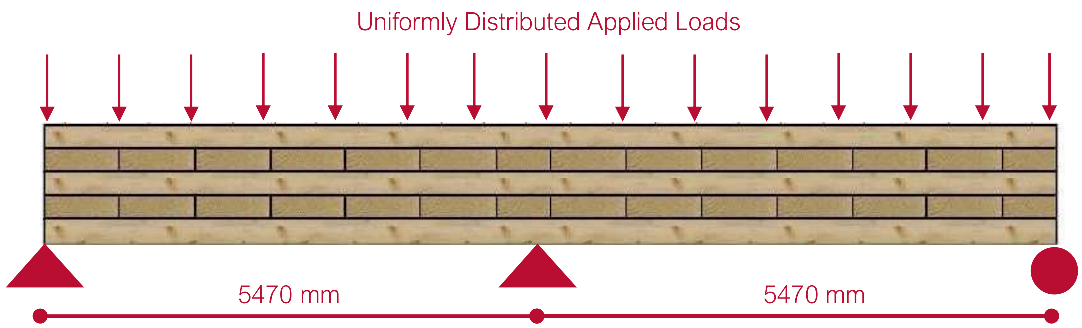


Figure 114: Red Stag CLT Panel Elevation

38.2 Assumption and Applied Loads:

Strength Reduction Factor (Ø) = 0.9 [36]

Bending Strength (F_b) = 14 MPa [36]

CLT Weight = 0.63 kPa - Calculated based on a Red Stag CLT density of 500 kg/m^3.

Additional Dead Load = 0.5 kPa

Live Load = 3.0 kPa - Refer to AS/NZS 1170.1 [37]

38.3 Calculation of the Effective Bending Stiffness using the Mechanical Jointed Beam Theory (Gamma Method)

L = Span of panels = 5470 mm = 5.47 m

b = Width of the CLT panel = 1 m [38]

h_i = Thickness of board layers in direction of action [38]

h_1 = 42 mm

h_2 = 42 mm

h_3 = 42 mm

$\overline{h_i}$ = Thickness of board layers in direction perpendicular to actions [38]

$\overline{h_1}$ = 42 mm

$\overline{h_2}$ = 42 mm

$A_i = b_i \times h_i$ [38]

$A_1 = (42\times1000) = 42000\ mm^2$

$A_2 = (42\times1000) = 42000\ mm^2$

$A_3 = (42\times1000) = 42000\ mm^2$

$I_i = \frac{b_i \times {h_i}^3}{12}$ [38]

$I_1 = \frac{b \times {h_1}^3}{12} = \frac{1000\times42^3}{12} = 6174000\ mm^4$

$I_2 = \frac{b \times {h_3}^3}{12} = \frac{1000\times42^3}{12} = 6174000\ mm^4$

$I_3 = \frac{b \times {h_3}^3}{12} = \frac{1000\times42^3}{12} = 6174000\ mm^4$

E_1 = 8000 MPa [36]

E_2 = 8000 MPa [36]

E_3 = 8000 MPa [36]

$\gamma_2 = 1$ [38]

$\gamma_1 = \frac{1}{1+\Pi^2 \times \frac{E_1 \times A_1}{L^2} \times \frac{\overline{h_1}}{G_R \times b}}$ [38]

$\gamma_3 = \frac{1}{1+\Pi^2 \times \frac{E_3 \times A_3}{L^2} \times \frac{\overline{h_2}}{G_R \times b}}$ [38]

where $\frac{h_i}{G_R \times b}$ = slip modulus due to shear deformation between layers and G_R = shear modulus perpendicular to the grain or rolling shear modulus [38].

E_0= MoE for longitudinal layers = 8000 MPa [36]	E_0= MoE for transvers layers = 6000 MPa [36]
E_{90} = 266.67 MPa	E_{90} = 200 MPa
G_0 = 500 MPa	G_0 = 375 MPa
G_R = 50 GPa [38]	G_R = 37.5 GPa [38]

L = span in mm (simple span; in direction of action //) [38]

$\gamma_1 = \frac{1}{1+\frac{3.14^2 \times 8000 \times 42000}{5470^2} \times \frac{42}{37.5 \times 1000}} = 0.890$

$\gamma_3 = \frac{1}{1+\frac{3.14^2 \times 8000 \times 42000}{5470^2} \times \frac{42}{37.5 \times 1000}} = 0.890$

$\overline{a_1} = \frac{h_1}{2} + \overline{h_1} + \frac{h_2}{2} - a_2$ [38]

$\overline{a_2} = \frac{h_2}{2} + \overline{h_2} + \frac{h_3}{2} - a_2$ [38]

$a_2 = 0$

$\overline{a_1} = \frac{42}{2} + 42 + \frac{42}{2} - 0 = 82$ mm

$\overline{a_2} = \frac{42}{2} + 42 + \frac{42}{2} - 0 = 82$ mm

$EI_{eff\,//} = \sum_{i=1}^{2} (E_i I_i + \gamma_i E_i A_i a_i^2)$ [38]

$EI_{eff\,//} = (E_1 I_1 + \gamma_1 E_1 A_1 a_1^2) + I_2 + (E_3 I_3 + \gamma_3 E_3 A_3 a_3^2)$ [38]

$EI_{eff} = 4.37\times10^{12}$ N.mm^2

$I_{eff} = \frac{4.37\times10^{12}}{8000} = 5.46 \times 10^8$ mm^4

38.4 Calculation of Bending Strength using the Mechanically Jointed Beams Theory (Gamma Method)

$M_r = Ø\times F_b\times\frac{I_{eff}}{(\delta_1 a_1+0.5h_1)}$ $(E_1 = E_2)$ [38]

F_b = 14 MPa [36]

Ø = 0.9 [36], [38]

$M_r = 0.9 \times 14 \times \frac{5.46 \times 10^8}{(0.890 \times 84+0.5\times42)} \times 10^{-6} = 71.84$ kN.m

38.5 Calculation of Bending Strength using the Simplified Method

$M_r = Ø \times F_b \times \frac{I_{eff}}{0.5h_1}$ [38]

F_b = 14 [36]

Ø = 0.9 [36], [38]

$M_r = 0.9 \times 14 \times \frac{5.46 \times 10^8}{0.5\times210}\times10^{-6} = 65.50$ kN.m

38.6 Calculation of Applied Bending Moment

$M^* = \frac{(1.2\times\text{Dead Load} \quad 1.5\times\text{Live Load})\times Span^2}{8}$ [38], [39]

$M^* = \frac{(1.35\times\text{Dead Load})\times Span^2}{8}$ [38], [39]

Dead Load = CLT Weight + Additional Dead Load

CLT Weight = CLT Thickness × Timber Weight = 210 × (5/1000) = 1.08 kPa.

Additional Dead Load = 0.5 kPa.

Live Load = 3 kPa on Floor

$M^* = \frac{(1.2\times(1.08+0.5)+ 1.5\times3)\times5.33^2}{8} = 22.59$ kN.m

$M^* = \frac{(1.35\times(1.08+0.5))\times5.33^2}{8} = 8.26$ kN.m

38.7 Bending Capacity Check

$M_{r\ \text{Mechanical jointed method}}$ = 71.84 kN.m ≥ M^*= 22.59 kN.m √ ok

$M_{r\ \text{Mechanical jointed method}}$ = 71.84 kN.m ≥ M^*= 8.26 kN.m √ ok

$M_{r\ \text{Simplified method}}$ = 65.50 kN.m ≥ M^*= 22.59 kN.m √ ok

$M_{r\ \text{Simplified method}}$ = 65.50 kN.m ≥ M^*= 8.26 kN.m √ ok

38.8 Deflection Check

$$\Delta_{\text{CLT Deflection}} = \frac{5\times(\text{Uniformly Distributed Applied Load})\times Span^4}{(384\times(EI_{eff})}$$ [38], [39]

$$\Delta_{\text{CLT Deflection}} = \frac{5\times((\text{CLT Weight + Additional Dead Load })+0.4\times\text{Live Load })\times 5470^4}{(384\times(EI_{eff}))}$$

$$\Delta_{\text{CLT Deflection}} = \frac{5\times((1.08+0.5)+0.4\times 3)\times 5470^4}{(384\times(4.37\times 10^{12})} = 6.66 \text{ mm}$$

Creep Factor (K_2) = 2

Long term deflection = $\frac{7.34}{2.4}\times 2$ = 6.12 mm → long term deflection

Long term deflection = 6.12 ≤ $\Delta^* = \frac{5470}{400}$ = 13.675 mm √ ok

38.9 Vibration Check

$$L \leq 0.11\,\frac{\left(\frac{(EI)_{eff}}{10^6}\right)^{0.293}}{m^{0.123}}$$ [41]

L = vibration -controlled span limit in m.

L = is the floor span in m.

m = linear mass of the CLT for a 1-m wide panel (kg/m).

EI_{eff} = effective bending stiffness.

$$L \leq 0.11\,\frac{\left(\frac{4.37\times 10^{12}}{10^6}\right)^{0.293}}{(1.0\times 0.210\times 500)^{0.123}} = 5.47 \text{ m} \geq 5.47 \text{ m}$$ √ ok

39.1 CLT Floor Panel Design – Longitudinal Direction

Calculation of the longitudinal members is based on the FPInnovation CLT design guide Mechanical jointed and simplified methods.

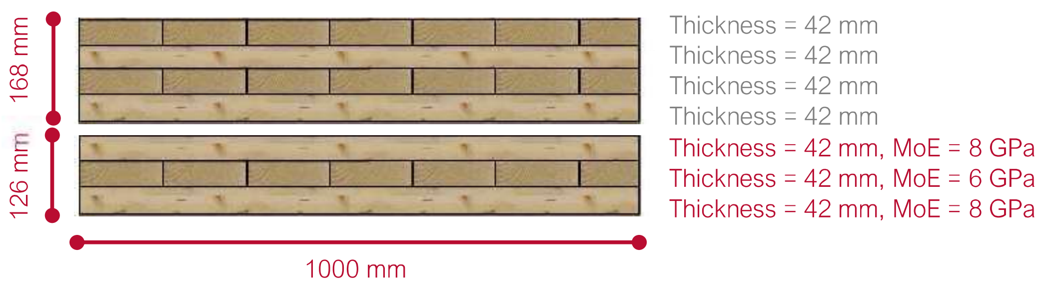


Figure 115: Red Stag CLT Panel Cross-Section

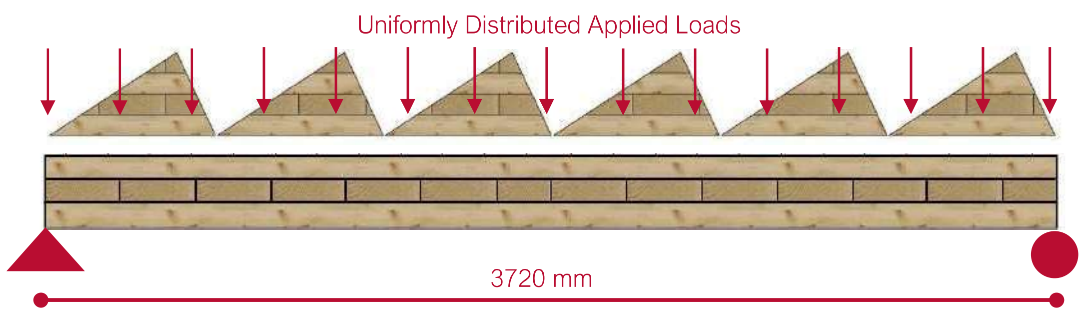


Figure 116: Red Stag CLT Panel Elevation

39.2 Assumption and Applied Loads:

Strength Reduction Factor (Ø) = 0.9 [36]

Bending Strength (F_b) = 14 MPa [36]

CLT Weight = 0.63 kPa - Calculated based on a Red Stag CLT density of 500 kg/m^3.

Additional Dead Load (Trade & Riser Weight) = 0.3 kPa

Live Load = 3.0 kPa - Refer to AS/NZS 1170.1 [37]

39.3 Calculation of the Effective Bending Stiffness using the Mechanical Jointed Beam Theory (Gamma Method)

L = Span of panels = 3850 mm = 3.72 m

b = Width of the CLT panel = 1 m [38]

h_i = Thickness of board layers in direction of action [38]

h_1 = 42 mm

h_2 = 42 mm

$\overline{h_i}$ = Thickness of board layers in direction perpendicular to actions [38]

$\overline{h_1}$ = 42 mm

$A_i = b_i \times h_i$ [38]

$A_1 = (42\times1000) = 42000\ \text{mm}^2$

$A_2 = (42\times1000) = 42000\ \text{mm}^2$

$I_1 = \frac{b_i \times {h_i}^3}{12}$ [38]

$I_1 = \frac{b \times {h_1}^3}{12} = \frac{1000\times42^3}{12} = 6174000\ \text{mm}^4$

$I_2 = \frac{b \times {h_3}^3}{12} = \frac{1000\times42^3}{12} = 6174000\ \text{mm}^4$

E_1 = 8000 MPa [36]

E_2 = 8000 MPa [36]

$\gamma_1 = \frac{1}{1+\Pi^2\times\frac{E_1\times A_1}{L^2}\times\frac{\overline{h_1}}{G_R\times b}}$ [38]

$\gamma_2 = \frac{1}{1+\Pi^2\times\frac{E_3\times A_3}{L^2}\times\frac{\overline{h_2}}{G_R\times b}}$ [38]

where $\frac{h_i}{G_R \times b}$ = slip modulus due to shear deformation between layers and G_R = shear modulus perpendicular to the grain or rolling shear modulus [38].

E_0= MoE for longitudinal layers = 8000 MPa [36]	E_0= MoE for transvers layers = 6000 MPa [36]
E_{90} = 266.67 MPa	E_{90} = 200 MPa
G_0 = 500 MPa	G_0 = 375 MPa
G_R = 50 GPa [38]	G_R = 37.5 GPa [38]

L = span in mm (simple span; in direction of action //) [38]

$$\gamma_1 = \frac{1}{1+\frac{3.14^2\times 8000\times 42000}{3720^2}\times\frac{42}{37.5\times 1000}} = 0.882$$

$$\gamma_2 = \frac{1}{1+\frac{3.14^2\times 8000\times 42000}{3720^2}\times\frac{42}{37.5\times 1000}} = 0.882$$

$\overline{a}_1 = \frac{h_1}{2} + \frac{h_1}{2}$ [38]

$\overline{a}_1 = \frac{h_2}{2} + \frac{h_1}{2}$ [38]

$\overline{a}_1 = \frac{42}{2} + \frac{42}{2}$ = 42 mm

$\overline{a}_2 = \frac{42}{2} + \frac{42}{2}$ = 42 mm

$EI_{eff\,//} = \sum_{i=1}^{2}(E_i I_i + \gamma_i E_i A_i {a_i}^2)$ [38]

$EI_{eff\,//} = (E_1 I_1 + \gamma_1 E_1 A_1 {a_1}^2) + (E_2 I_2 + \gamma_2 E_2 A_2 {a_2}^2)$ [38]

$EI_{eff}=(8000\times 6174000+0.882\times 8000\times 42000\times 42^2)+(8000\times 6174000+0.882\times 42000\times 8000\times 42^2)$

$= 5.72\times 10^{11}+5.72\times 10^{11}= 1.145\times 10^{12}$ N.mm^2

$I_{eff} = \frac{1.145\times 10^{12}}{8000} = 1.43 \times 10^8$ mm^4

39.4 Calculation of Bending Strength using the Mechanically Jointed Beams Theory (Gamma Method)

$M_r = Ø \times F_b \times \frac{I_{eff}}{(\delta_1 a_1 + 0.5h_1)}$ $(E_1 = E_2)$ [38]

F_b = 14 MPa [36]

Ø = 0.9 [36], [38]

$M_r = 0.9 \times 14 \times \frac{1.145 \times 10^8}{(0.882 \times 42 + 0.5 \times 42)} \times 10^{-6} = 31.05$ kN.m

39.5 Calculation of Bending Strength using the Simplified Method

$M_r = Ø \times F_b \times \frac{I_{eff}}{0.5h_1}$ [38]

F_b = 14 [36]

Ø = 0.9 [36], [38]

$M_r = 0.9 \times 14 \times \frac{1.43 \times 10^8}{0.5 \times 126} \times 10^{-6} = 28.63$ kN.m

39.6 Calculation of Applied Bending Moment

$M^* = \frac{(1.2 \times \text{Dead Load} \quad 1.5 \times \text{Live Load}) \times Span^2}{8}$ [38], [39]

$M^* = \frac{(1.35 \times \text{Dead Load}) \times Span^2}{8}$ [38], [39]

Dead Load = CLT Weight + Additional Dead Load

CLT Weight = CLT Thickness × Timber Weight = 126 × (5/1000) = 0.63 kPa.

Additional Dead Load = 0.5 kPa.

Live Load = 3 kPa on Stair Stringer

$M^* = \frac{(1.2 \times (0.63 + 0.3) + 1.5 \times 3) \times 3.725^2}{8} = 9.77$ kN.m

$M^* = \frac{(1.35 \times (0.63 + 0.3)) \times 3.725^2}{8} = 2.43$ kN.m

39.7 Bending Capacity Check

$M_{r\ \text{Mechanical jointed method}}$ = 31.05 kN.m ≥ M^*= 9.77 kN.m √ ok

$M_{r\ \text{Mechanical jointed method}}$ = 31.05 kN.m ≥ M^*= 2.43 kN.m √ ok

$M_{r\ \text{Simplified method}}$ = 28.62 kN.m ≥ M^*= 9.77 kN.m √ ok

$M_{r\ \text{Simplified method}}$ = 28.62 kN.m ≥ M^*= 2.43 kN.m √ ok

39.8 Deflection Check

$$\Delta_{\text{CLT Deflection}} = \frac{5\times(\text{Uniformly Distributed Applied Load})\times Span^4}{(384\times(EI_{eff})}$$ [38], [39]

$$\Delta_{\text{CLT Deflection}} = \frac{5\times((\text{CLT Weight + Additional Dead Load })+0.4\times\text{Live Load })\times 3720^4}{(384\times(EI_{eff}))}$$

$$\Delta_{\text{CLT Deflection}} = \frac{5\times((0.63+0.3)+0.4\times 3)\times 3720^4}{(384\times(1.145\times 10^{12}\ )} = 4.64 \text{ mm}$$

Creep Factor (K_2) = 2

Long term deflection = 4.64 × 2 = 9.28 mm → long term deflection

Long term deflection = 9.28 ≤ $\Delta^* = \frac{3720}{400}$ = 9.30 mm √ ok

39.9 Vibration Check

$$L \leq 0.11\frac{(\frac{(EI)_{eff}}{10^6})^{0.293}}{\text{m}^{0.123}}$$

L = vibration -controlled span limit in m.
m = linear mass of the CLT for a 1-m wide panel (kg/m).
EI_{eff} = effective bending stiffness.

$$L \leq 0.11\frac{(\frac{1.145\times 10^{12}}{10^6})^{0.293}}{(1.0\times 0.126\times 500)^{0.123}} = 3.94 \text{ m}$$

Vibration span = 3.94 ≥ Maximum length of the CLT panels = 3.72 m √ ok

40.1 CLT Roof Panel Design – Longitudinal Direction

Calculation of the longitudinal members is based on the FPInnovation CLT design guide Mechanical jointed and simplified methods.

Figure 117: Red Stag CLT Panel Cross-Section

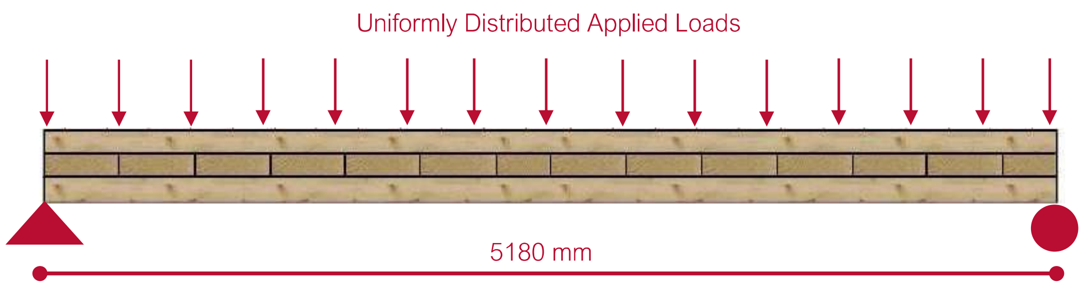


Figure 118: Red Stag CLT Panel Elevation

40.2 Assumption and Applied Loads:

Strength Reduction Factor (Ø) = 0.9 [36]

Bending Strength (F_b) = 14 MPa [36]

CLT Weight = 0.63 kPa - Calculated based on a Red Stag CLT density of 500 kg/m^3.

Additional Dead Load = 0.1 kPa

Live Load = 0.25 kPa - Refer to AS/NZS 1170.1 [37]

40.3 Calculation of the Effective Bending Stiffness using the Mechanical Jointed Beam Theory (Gamma Method)

L = Span of panels = 5180 mm = 5.18 m

b = Width of the CLT panel = 1 m [38]

h_i = Thickness of board layers in direction of action [38]

h_1 = 42 mm

h_2 = 42 mm

$\overline{h_i}$ = Thickness of board layers in direction perpendicular to actions [38]

$\overline{h_1}$ = 42 mm

$A_i = b_i \times h_i$ [38]

A_1 = (42×1000) = 42000 mm^2

A_2 = (42×1000) = 42000 mm^2

$I_1 = \frac{b_i \times {h_i}^3}{12}$ [38]

$I_1 = \frac{b \times {h_1}^3}{12} = \frac{1000 \times 42^3}{12}$ = 6174000 mm^4

$I_2 = \frac{b \times {h_3}^3}{12} = \frac{1000 \times 42^3}{12}$ = 6174000 mm^4

E_1 = 8000 MPa [36]

E_2 = 8000 MPa [36]

$\gamma_1 = \frac{1}{1+\Pi^2 \times \frac{E_1 \times A_1}{L^2} \times \frac{\overline{h_1}}{G_R \times b}}$ [38]

$\gamma_2 = \frac{1}{1+\Pi^2 \times \frac{E_3 \times A_3}{L^2} \times \frac{\overline{h_2}}{G_R \times b}}$ [38]

where $\frac{h_i}{G_R \times b}$ = slip modulus due to shear deformation between layers and G_R = shear modulus perpendicular to the grain or rolling shear modulus [38].

E_0= MoE for longitudinal layers = 8000 MPa [36]	E_0= MoE for transvers layers = 6000 MPa [36]
E_{90} = 266.67 MPa	E_{90} = 200 MPa
G_0 = 500 MPa	G_0 = 375 MPa
G_R = 50 GPa [38]	G_R = 37.5 GPa [38]

L = span in mm (simple span; in direction of action //) [38]

$ɣ_1 = \frac{1}{1+\frac{3.14^2 \times 8000 \times 42000}{5180^2} \times \frac{42}{37.5 \times 1000}} = 0.935$

$ɣ_2 = \frac{1}{1+\frac{3.14^2 \times 8000 \times 42000}{5180^2} \times \frac{42}{37.5 \times 1000}} = 0.935$

$\overline{a}_1 = \frac{h_1}{2} + \frac{h_1}{2}$ [38]

$\overline{a}_1 = \frac{h_2}{2} + \frac{h_1}{2}$ [38]

$\overline{a}_1 = \frac{42}{2} + \frac{42}{2} = 42$ mm

$\overline{a}_2 = \frac{42}{2} + \frac{42}{2} = 42$ mm

$EI_{eff\,//} = \sum_{i=1}^{2}(E_i I_i + ɣ_i E_i A_i a_i{}^2)$ [38]

$EI_{eff\,//} = (E_1 I_1 + ɣ_1 E_1 A_1 a_1{}^2) + (E_2 I_2 + ɣ_2 E_2 A_2 a_2{}^2)$ [38]

$EI_{eff} = (8000 \times 6174000 + 0.935 \times 8000 \times 42000 \times 42^2) + (8000 \times 6174000 + 0.935 \times 42000 \times 8000 \times 42^2)$

$= 6.038 \times 10^{11} + 6.038 \times 10^{11} = 1.207 \times 10^{12}$ N.mm^2

$I_{eff} = \frac{1.207 \times 10^{12}}{8000} = 1.509 \times 10^8$ mm^4

40.4 Calculation of Bending Strength using the Mechanically Jointed Beams Theory (Gamma Method)

$M_r = Ø \times F_b \times \frac{I_{eff}}{(\delta_1 a_1 + 0.5 h_1)}$ $(E_1 = E_2)$ [38]

$F_b = 14$ MPa [36]

$Ø = 0.9$ [36], [38]

$M_r = 0.9 \times 14 \times \frac{1.509 \times 10^8}{(0.935 \times 42 + 0.5 \times 42)} \times 10^{-6} = 31.55$ kN.m

40.5 Calculation of Bending Strength using the Simplified Method

$M_r = Ø \times F_b \times \frac{I_{eff}}{0.5h_1}$ [38]

$F_b = 14$ [36]

$Ø = 0.9$ [36], [38]

$M_r = 0.9 \times 14 \times \frac{1.509 \times 10^8}{0.5 \times 126} \times 10^{-6} = 30.19$ kN.m

40.6 Calculation of Applied Bending Moment

$M^* = \frac{(1.2 \times \text{Dead Load} + 1.5 \times \text{Live Load}) \times Span^2}{8}$ [38], [39]

$M^* = \frac{(1.35 \times \text{Dead Load}) \times Span^2}{8}$ [38], [39]

Dead Load = CLT Weight + Additional Dead Load

CLT Weight = CLT Thickness × Timber Weight = 126 × (5/1000) = 0.63 kPa.

Additional Dead Load = 0.1 kPa.

Live Load = 0.25 kPa on Roof

$M^* = \frac{(1.2 \times (0.63+0.1) + 1.5 \times 0.25) \times 5180^2}{8} = 4.20$ kN.m

$M^* = \frac{(1.35 \times (0.63+0.1)) \times 5180^2}{8} = 3.67$ kN.m

40.7 Bending Capacity Check

$M_{r\ \text{Mechanical jointed method}}$ = 31.55 kN.m ≥ M^*= 4.20 kN.m √ ok

$M_{r\ \text{Mechanical jointed method}}$ = 31.55 kN.m ≥ M^*= 3.67 kN.m √ ok

$M_{r\ \text{Simplified method}}$ = 30.19 kN.m ≥ M^*= 4.20 kN.m √ ok

$M_{r\ \text{Simplified method}}$ = 30.19 kN.m ≥ M^*= 3.67 kN.m √ ok

40.8 Deflection Check

$\Delta_{\text{CLT Deflection}} = \frac{5\times(\text{Uniformly Distributed Applied Load})\times Span^4}{(384\times(EI_{eff})}$ [38], [39]

$\Delta_{\text{CLT Deflection}} = \frac{5\times((\text{CLT Weight} + \text{Additional Dead Load})+0.4\times\text{Live Load})\times 8150^4}{(384\times(EI_{eff}))}$

$\Delta_{\text{CLT Deflection}} = \frac{5\times((0.63+0.1)+0.4\times0.25)\times5180^4}{(384\times(1.207\times10^{12})}$ = 6.44 mm

Creep Factor (K_2) = 2

Long term deflection = 6.44 × 2 = 12.89 mm → long term deflection

Long term deflection = 12.89 **≤** $\Delta^* = \frac{5180}{400}$ = 12.95 mm √ **ok**

41. Three Layer Double Span CLT Roof Design Calculation Example

41.1 CLT Roof Panel Design – Longitudinal Direction

Calculation of the longitudinal members is based on the FPInnovation CLT design guide Mechanical jointed and simplified methods.

Figure 119: Red Stag CLT Panel Cross-Section

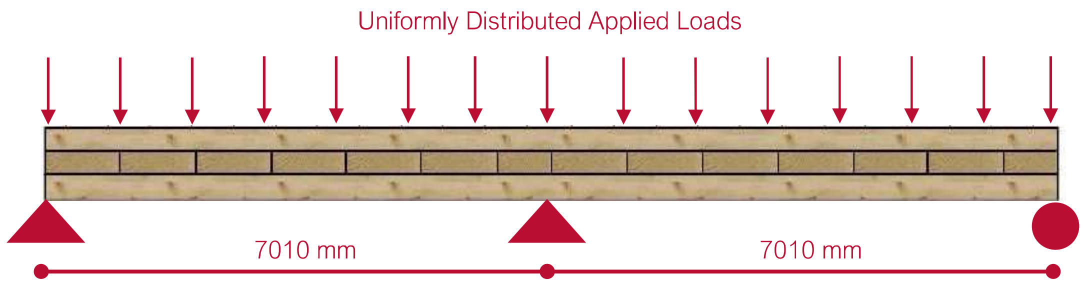


Figure 120: Red Stag CLT Panel Elevation

41.2 Assumption and Applied Loads:

Strength Reduction Factor (Ø) = 0.9 [36]

Bending Strength (F_b) = 14 MPa [36]

CLT Weight = 0.63 kPa - Calculated based on a Red Stag CLT density of 500 kg/m^3.

Additional Dead Load = 0.1 kPa

Live Load = 0.25 kPa - Refer to AS/NZS 1170.1 [7]

41.3 Calculation of the Effective Bending Stiffness using the Mechanical Jointed Beam Theory (Gamma Method)

L = Span of panels = 7010 mm = 7.01 m

b = Width of the CLT panel = 1 m [38]

h_i = Thickness of board layers in direction of action [38]

h_1 = 42 mm

h_2 = 42 mm

$\overline{h_i}$ = Thickness of board layers in direction perpendicular to actions [38]

$\overline{h_1}$ = 42 mm

$A_i = b_i \times h_i$ [38]

A_1 = (42×1000) = 42000 mm^2

A_2 = (42×1000) = 42000 mm^2

$I_1 = \frac{b_i \times {h_i}^3}{12}$ [38]

$I_1 = \frac{b \times {h_1}^3}{12} = \frac{1000 \times 42^3}{12}$ = 6174000 mm^2

$I_2 = \frac{b \times {h_3}^3}{12} = \frac{1000 \times 42^3}{12}$ = 6174000 mm^2

E_1 = 8000 MPa [36]

E_2 = 8000 MPa [36]

$\gamma_1 = \frac{1}{1+\Pi^2 \times \frac{E_1 \times A_1}{L^2} \times \frac{\overline{h_1}}{G_R \times b}}$ [38]

$\gamma_2 = \frac{1}{1+\Pi^2 \times \frac{E_3 \times A_3}{L^2} \times \frac{\overline{h_2}}{G_R \times b}}$ [38]

where $\frac{h_i}{G_R \times b}$ = slip modulus due to shear deformation between layers and G_R = shear modulus perpendicular to the grain or rolling shear modulus [38].

E_0= MoE for longitudinal layers = 8000 MPa [36]	E_0= MoE for transvers layers = 6000 MPa [36]
E_{90} = 266.67 MPa	E_{90} = 200 MPa
G_0 = 500 MPa	G_0 = 375 MPa
G_R = 50 GPa [38]	G_R = 37.5 GPa [38]

L = span in mm (simple span; in direction of action //) [38]

$\gamma_1 = \frac{1}{1+\frac{3.14^2 \times 8000 \times 42000}{7010^2} \times \frac{42}{37.5 \times 1000}}$ = 0.9636

$\gamma_2 = \frac{1}{1+\frac{3.14^2 \times 8000 \times 42000}{7010^2} \times \frac{42}{37.5 \times 1000}}$ = 0.9636

$a_1 = \frac{h_1}{2} + \frac{\overline{h_1}}{\underline{2}}$ [38]

$a_1 = \frac{h_2}{2} + \frac{h_1}{2}$ [38]

$a_1 = \frac{42}{2} + \frac{42}{2} = 42$ mm

$a_2 = \frac{42}{2} + \frac{42}{2} = 42$ mm

$EI_{eff\,//} = \sum_{i=1}^{2} (E_i I_i + \gamma_i E_i A_i {a_i}^2)$ [38]

$EI_{eff\,//} = (E_1 I_1 + \gamma_1 E_1 A_1 {a_1}^2) + (E_2 I_2 + \gamma_2 E_2 A_2 {a_2}^2)$ [38]

$EI_{eff} = (8000 \times 6174000 + 0.9636 \times 8000 \times 42000 \times 42^2) + (8000 \times 6174000 + 0.9636 \times 8000 \times 42000 \times 42^2)$

$= 1.241 \times 10^{12}$ N.mm^2

$I_{eff} = \frac{1.241 \times 10^{12}}{8000} = 1.55 \times 10^8$ mm^4

41.4 Calculation of Bending Strength using the Mechanically Jointed Beams Theory (Gamma Method)

$M_r = \varnothing \times F_b \times \frac{I_{eff}}{(\delta_1 a_1 + 0.5 h_1)}$ $(E_1 = E_2)$ [38]

$F_b = 14$ MPa [36]

$\varnothing = 0.9$ [36], [38]

$M_r = 0.9 \times 14 \times \frac{1.51 \times 10^8}{(0.9636 \times 42 + 0.5 \times 42)} \times 10^{-6} = 31.55$ kN.m

41.5 Calculation of Bending Strength using the Simplified Method

$M_r = \varnothing \times F_b \times \frac{I_{eff}}{0.5 h_1}$ [38]

$F_b = 14$ [36]

$\varnothing = 0.9$ [36], [38]

$M_r = 0.9 \times 14 \times \frac{1.55 \times 10^8}{0.5 \times 126} \times 10^{-6} = 31.80$ kN.m

41.6 Calculation of Applied Bending Moment

$M^{*}=\frac{(1.2\times\text{Dead Load}\quad 1.5\times\text{Live Load})\times Span^{2}}{8}$ [38], [39]

$M^{*}=\frac{(1.35\times\text{Dead Load})\times Span^{2}}{8}$ [38], [39]

Dead Load = CLT Weight + Additional Dead Load

CLT Weight = CLT Thickness × Timber Weight = 126 × (5/1000) = 0.63 kPa.

Additional Dead Load = 0.1 kPa.

Live Load = 0.25 kPa on Roof

$M^{*}=\frac{(1.2\times(0.63+0.1)+1.5\times0.25)\times7.01^{2}}{8}$ = 7.68 kN.m

$M^{*}=\frac{(1.35\times(0.63+0.1))\times7.01^{2}}{8}$ = 6.73 kN.m

41.7 Bending Capacity Check

$M_{r\ \text{Mechanical jointed method}}$ = 31.80 kN.m ≥ M^{*}= 7.68 kN.m √ ok

$M_{r\ \text{Mechanical jointed method}}$ = 31.80 kN.m ≥ M^{*}= 6.73 kN.m √ ok

$M_{r\ \text{Simplified method}}$ = 31.03 kN.m ≥ M^{*}= 7.68 kN.m √ ok

$M_{r\ \text{Simplified method}}$ = 31.03 kN.m ≥ M^{*}= 6.73 kN.m √ ok

41.8 Deflection Check

$\Delta_{\text{CLT Deflection}}=\frac{5\times(\text{Uniformly Distributed Applied Load})\times Span^{4}}{(384\times(EI_{eff}\ )}$ [38], [39]

$\Delta_{\text{CLT Deflection}}=\frac{5\times((\text{CLT Weight}+\text{Additional Dead Load})+0.4\times\text{Live Load})\times7010^{4}}{(384\times(EI_{eff}))}$

$\Delta_{\text{CLT Deflection}}=\frac{5\times((0.63+0.1)+0.4\times0.25)\times7010^{4}}{(384\times(1.241\times10^{12}\ )}$ = 21.03 mm

Creep Factor (K_2) = 2

Long term deflection = $\frac{21.03}{2.4}\times 2$ = 17.52 mm → Long term deflection

Long term deflection = 17.52 mm ≤ $\Delta^{*}=\frac{7010}{400}$ = 17.52 mm √ ok